Forecasting GDP over the business cycle in a multi-frequency and data-rich environment

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Abstract

This paper merges two specifications recently developed in the forecasting literature: the MS-MIDAS model (Guérin and Marcellino, 2013) and the factor-MIDAS model (Marcellino and Schumacher, 2010). The MS-factor MIDAS model that we introduce incorporates the information provided by a large dataset consisting of mixed frequency variables and captures regime-switching behaviors. Monte Carlo simulations show that this specification tracks the dynamics of the process and predicts the regime switches successfully, both in-sample and out-of-sample. We apply this model to US data from 1959 to 2010 and properly detect recessions by exploiting the link between GDP growth and higher frequency financial variables.

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Introduction

The recent financial crisis has heightened practitioners' interest in models that differentiate GDP dynamics over the course of the business cycle, as that first initiated by Hamilton [1989]. In order to forecast GDP dynamics, macroeconomists are able to mobilize a very large set of indicators, as suggested by Stock and Watson [2005]. In this context, extracting common factors reflecting the co-movements of these indicators has proved to be a convenient way of summarizing the information contained in such large datasets. These indicators are very often available at higher frequencies than the targeted variable (GDP). This aggregation issue is quite successfully dealt with by MIxed DAta Sampling (MIDAS) models introduced by Ghysels, Santa-Clara and Valkanov [2004] and Ghysels, Sinko and Valkanov [2007]. This paper is at the crossroad of these three strands of the literature.

MIDAS models are regressions that involve variables sampled at different frequencies. In this framework, a low frequency variable can be explained by higher frequency indicators without any time aggregation procedure. A distributed lagged function can be used to obtain a parsimonious specification of the relationship between the dependent variable and the higher frequency variables. While MIDAS models were first applied to financial data[1], they have also become a popular tool for forecasting macroeconomic variables such as GDP growth. Forecasters use specifications that link the GDP variable to a handful of monthly leading indicators or they rely on combinations of MIDAS models to deal with the potentially large number of indicators.[2] See Andreou, Ghysels and Kourtellos [2011] for a survey of this literature.

Two recent extensions are particularly designed to forecast macroeconomic variables: factor MIDAS (FaMIDAS) models by Marcellino and Schumacher [2010] and Markov-Switching MIDAS models by Guérin and Marcellino [2013]. In addition to involving mixed frequency data, the first class of models allows the use of information provided by a large dataset and can handle unbalanced samples that practitioners usually have to work with because of different publication lags. The second class of models incorporates regime changes in the parameters of the relationship between the low and high frequency

[1]See Ghysels, Santa-Clara and Valkanov [2005], Ghysels, Santa-Clara and Valkanov [2006] and Ghysels et al. [2007] for applications to equity returns, Clements, Galvão and Kim [2008] to exchange rates.

[2]For instance, see Clements and Galvão [2008], Clements and Galvão [2009], Bai, Ghysels and Wright [2013], Armesto, Engemann and Owyang [2010a], Armesto, Hernández-Murillo, Owyang and Piger [2010b], Andreou, Ghysels and Kourtellos [2013] for US GDP, Kuzin, Marcellino and Schumacher [2011] for euro area GDP, Foroni, Marcellino and Schumacher [2012] for both US and euro area GDP.

variables. Moreover, it gives qualitative information about the state of the economy. This provides a useful tool for business cycle analysis.

In this paper, we introduce the MS Factor MIDAS model (MS-FaMIDAS), which captures both co-movements and regime shifts in the dynamics of the variables and which can be implemented with mixed frequency data. We consider the dynamic factor model of Giannone, Reichlin and Small [2008], estimated with the 2-step method of Doz, Giannone and Reichlin [2011]. This approach can deal with the unbalanced data availability at the end of the sample caused by uneven publication lags. It should be noted that we allow a switch on the coefficients of the equation of the dependent variable (the coefficients of the GDP equation in our application), but not on the factor dynamics.

The MS-FaMIDAS model helps improving the short-run analysis of business cycle fluctuations. It provides both quantitative information (GDP growth rate) and qualitative information (state of the economy). The MIDAS specification allows incorporating within-quarter information to update directly the GDP forecast and the probability of recession several times during that very quarter. Extracting factor from a larger set of indicators provides less noisy indicators and might improve the inference about the business cycle. Moreover, this approach overcomes the problem of missing data, as it can be implemented even when some observations are missing at the end of the sample because of publication lags. Finally, it can also deal with the irregular pattern of missing observations (the so-called *ragged edge* problem) caused by the different time releases of the indicators.

An obvious limitation of the model is its omission of potential switches in the factor representation. It could be relevant to incorporate regime switches in the factor dynamics, as done in Camacho, Perez-Quiros and Poncela [2010] and Camacho, Perez-Quiros and Poncela [2012]. There could also be a temporal instability in the factor loadings. Nevertheless, the recent literature provides theoretical and empirical evidence supporting the approach adopted in this paper. The presence of limited continuous time variation in the loadings or a few limited discrete jumps in the factor representation has a limited impact on estimation of the factor space. Factor-based forecasts are also found robust to empirically relevant forms of instability (see Banerjee, Marcellino and Masten [2008], Stock and Watson [2009] and Bates, Plagborg-Moller, Stock and Watson [2013]). In this paper, we provide new results in the case of Markov-switching in the factor representation. A Monte Carlo study in finite sample suggests that the omission of regime switches in the factor loadings or in the factor dynamics has a limited impact on the estimation and forecast of the dependent variable and the state.

We also use Monte Carlo experiments to assess the performance of the MS-FaMIDAS model relative to several benchmarks, both in-sample and out-of-sample. Particular attention is paid to the models' shortfalls resulting from the fact that regime switches and mixed frequency data have not been taken into account. We also compare MS-FaMIDAS models based on distributed lag polynomials with the unconstrained MIDAS models, as done in simple MIDAS models by Foroni et al. [2012], in FaMIDAS models in Marcellino and Schumacher [2010] and in MS-MIDAS models in Barsoum and Stankiewicz [2013]. The unrestricted variant is more flexible but far less parsimonious. The models are assessed for various sets of model parameters. In the out-of-sample evaluation, we use unbalanced datasets to take account of the uneven time releases of short-term indicators. Forecasting is also performed using direct and iterative methods and the results of the two approaches are compared. To our knowledge, this is the first paper comparing these two approaches in forecasting with MIDAS models.

The Monte Carlo simulations show that the new specification tracks the dynamics of the process relatively accurately and captures regime switches successfully. In contrast, there is a loss in the specifications that omit parameter changes or time aggregate higher frequency data to match the sampling rate of the lower frequency dependent variable. The unconstrained MS-FaMIDAS model is a serious competitor despite the proliferation of parameters when the lag increases. This last result is consistent with the findings of Marcellino and Schumacher [2010], Foroni et al. [2012] and Barsoum and Stankiewicz [2013], and may be due to the small difference in data frequencies in our paper as in typical macroeconomic applications.

We apply the MS-FaMIDAS to model the link between US GDP and financial variables sampled at a higher frequency. By doing so, we extend the empirical work conducted by Guérin and Marcellino [2013]. The authors use the MS-MIDAS specification to assess the predictive power for US GDP growth of three financial variables taken separately: the yield curve, the S&P500 index and the Federal Funds. In this paper, we use the block of financial variables considered in Stock and Watson [2005]. The dataset consists of money and credit quantity aggregates, stock prices, interest rates and spreads, exchange rates and price indices. A real-time evaluation shows that the MS-FaMIDAS model, with factors extracted from this dataset, properly detects the US recessions at horizons up to two quarters. In addition, our financial factors help to predict quantitatively the US GDP in the short run.

The remainder of this paper is organized as follows. In section 1, we present the MS-factor MIDAS specification and describe the estimation and forecasting techniques. In section 2, we use Monte Carlo simulations to assess the in-sample and out-of-sample performance of the specification. We also assess the impact on the estimation and forecast of the omission of potential Markov-switching in the factor representation. Section 3 is devoted to the empirical application to US data. The last section offers some concluding remarks.

1 A MS Factor MIDAS model

1.1 Specification

This section presents the MS Factor MIDAS model. We follow the notations of Clements and Galvão [2008] and Clements and Galvão [2009]. The time index t denotes the time unit of the lower frequency variable Y (a quarter in our application). We model the link between Y and higher frequency indicators X sampled m times between two time units of Y, e.g. t and $t-1$ ($m = 3$ for monthly indicators as in our application). The lag operator $L^{1/m}$ operates at the higher frequency, e.g. $L^{s/m}x_t^{(m)} = x_{t-s/m}^{(m)}$.

Consider a vector of N stationary monthly series $X_t^{(m)} = \left(X_{1t}^{(m)}, X_{2t}^{(m)}, \ldots, X_{Nt}^{(m)}\right)'$, $t = 1/m, 2/m, \ldots, T$ previously standardized to mean zero and variance one. We assume that the observed variables $X_t^{(m)}$ can be broken down into the sum of two unobserved orthogonal components: a small number of latent variables, with the common factors f_t summarizing the dynamics common to all the series, and an idiosyncratic component ε_t, specific to each series. In addition, the factors can be autocorrelated. Formally, the dynamic factor model is given by:

$$X_t^{(m)} = \Lambda f_t^{(m)} + \varepsilon_t^{(m)} \qquad \varepsilon_t^{(m)} i.i.d.N(0, \Sigma_\varepsilon) \tag{1}$$

$$f_t^{(m)} = A_1 f_{t-1/m}^{(m)} + \ldots + A_p f_{t-p/m}^{(m)} + B u_t^{(m)} \qquad u_t^{(m)} i.i.d.N(0, I_q) \tag{2}$$

for $t = 1/m, 2/m, \ldots, T$. In equation (1), $f_t^{(m)} = (f_{1t}^{(m)}, \ldots, f_{rt}^{(m)})'$ is an $(r \times 1)$ stationary process, Λ is an $(N \times r)$ matrix of factor loadings, $\varepsilon_t^{(m)} = (\varepsilon_{1t}^{(m)}, \ldots, \varepsilon_{Nt}^{(m)})'$ is an $N \times 1$ stationary process, $(f_t^{(m)})$ and $(\varepsilon_t^{(m)})$ are independent processes. In equation (2), the VAR process of $f_t^{(m)}$ is driven by a q-dimensional standardized white noise $u_t^{(m)}$ (the dynamic shocks), $A_1, \ldots, A_p$ are $(r \times r)$ matrices of parameters and B is an $(r \times q)$ matrix.

The system of equations (1)-(2) can be cast in a state space representation. The measurement equation (1) describes the relationship between the observed variable $X_t^{(m)}$ and the unobserved state variable $f_t^{(m)}$. The state equation (2) describes how the factors are generated from their lags and from innovations.

The information summarized in the latent factors is then used to forecast the lower frequency variable y_t. To relate the variable y_t to the higher frequency factors, Marcellino and Schumacher [2010] introduce the Factor MIDAS model given by:

$$y_t = \beta_0 + \beta_1 B(L^{1/m}, \theta) f_t^{(m)} + \eta_t \qquad t = 1, \ldots, T \tag{3}$$

where $f_t^{(m)}$ is a latent factor. The superscript (m) indicates that this variable is sampled at a higher frequency.

The polynomial $B(L^{1/m}, \theta)$ is the exponential Almon lag[3] with:

$$B(L^{1/m}, \theta) = \sum_{j=1}^{K} b(j, \theta) L^{(j-1)/m}, b(j, \theta) = \frac{exp(\theta_1 j + \theta_2 j^2)}{\sum_{j=1}^{K} exp(\theta_1 j + \theta_2 j^2)} \tag{4}$$

This function implies that the weights are positive. It allows a parsimonious specification since only two coefficients are needed for the K lags. The coefficient $\theta = \{\theta_1, \theta_2\}$ defines the lag structure and the coefficient β_1 in equation (3) gives the final impact of the factor on the dependent variable. In the particular case where $\theta = \{0, 0\}$, we obtain the standard equal weighting aggregation scheme. If $\theta_2 \leq 0$, the weight decreases with the lag j.

For r factors, the specification is given by:

$$y_t = \beta_0 + \sum_{i=1}^{r} \beta_{1,i} B(L^{1/m}, \theta_i) f_{i,t}^{(m)} + \varepsilon_t \qquad t = 1, \ldots, T \tag{5}$$

It should be noted that the lag structure can be different for each factor. This is particularly relevant for GDP forecasting, using a large spectrum of explanatory variables. Indeed, a larger weight should be attached to the lagged values of a factor extracted from leading indicators (such as financial data) and to more contemporaneous values of a factor

[3] Other possible specifications of the MIDAS polynomials are based on beta or step functions. See Ghysels et al. [2007] for a presentation of the various parameterizations of $B(L^{1/m}, \theta)$.

extracted from coincident indicators (such as survey data or GDP components).

Like Guérin and Marcellino [2013], we extend the specification of y_t by allowing a change in the parameters of the model. We assume that the parameters of equation (5) depend on an unobservable discrete variable S_t:

$$y_t = \beta_0(S_t) + \sum_{i=1}^{r} \beta_{1,i}(S_t) B(L^{1/m}, \theta_i) f_{i,t}^{(m)} + \varepsilon_t(S_t) \qquad t = 1, \ldots, T \tag{6}$$

where $\varepsilon_t | S_t \to NID(0, \sigma^2(S_t))$. Note that the lag structure $B(L^{1/m}, \theta)$ is not regime-dependent. This is a strong assumption but allowing a change in the weight parameters would additionally increase the computational burden. The variable $S_t = 1, 2, \ldots, M$ represents the state that the process is in at time t. This variable is assumed to follow a first-order Markov chain defined by the following transition probabilities:

$$p_{ij} = P(S_t = j | S_{t-1} = i) \tag{7}$$

where $\sum_{j=1}^{M} p_{ij} = 1$, $\forall i, j = 1, 2, \ldots, M$.

It should be noted that it is also possible to deal with the mixed frequencies in the state space representation of the factor model, as done by Bańbura and Rünstler [2011]. The authors modify the state representation of the factor model to include monthly GDP growth as a latent variable in the state vector. Several papers discuss the connection between the two approaches. From a theoretical point of view, Bai et al. [2013] show that, in some cases, the MIDAS representation is an exact representation of the state space approach and, in other cases, it involves approximation errors that are typically small. The empirical comparison of the two approaches by Marcellino and Schumacher [2010] and Kuzin et al. [2011] shows that the MIDAS approach, which is more parsimonious and less prone to specification errors, performs quite well. In this paper, we do not use the integrated state space approach, which appears more complicated with regime-switching parameters.

1.2 Estimation

The estimation of the MS-FaMIDAS model consists of two main steps. We first estimate the factors. At this level, we use a method that copes with unbalanced datasets possibly resulting from different publication lags of the higher frequency indicators. We then estimate the relationship between the low frequency variable and the high frequency factors.

1. Estimation of the factors (equations 1-2): we apply the two-step method proposed by Doz et al. [2011] to estimate the factors at a monthly frequency. Factors are first estimated by principal components on the balanced sub-sample, i.e. over the period when all the variables X_t are known. The factors are then estimated over the entire range of observations including the period when some variables have missing observations. At this stage, we apply the Kalman filter and smoother to the state space representation. To accommodate the missing observations at the end of the sample caused by publication lags, the variance of the idiosyncratic noise related to the missing observations is set to infinity (this is equivalent to skipping these observations).

2. Estimation of the MS-model (equations 6-7): like Guérin and Marcellino [2013], we estimate equations (6)-(7) via maximum likelihood. The likelihood is derived in the Hamilton filter and the simplex search method is applied to find the vector of parameters that maximizes the function (we use Matlab's *fminsearch* function). A smoothing algorithm is then applied to obtain a better estimation of the states. In the estimation procedure, the parameter θ_2 of the Almon function is constrained to be negative, which guarantees a declining weight of the factors as the lag length increases (see for instance Ghysels et al. [2007] for a further discussion of this point).

1.3 Forecast

Once the specification has been estimated, it can be used to derive a forecast of y_t and S_t. We consider two alternative approaches, known in the forecasting literature as the iterative and direct approaches.[4]

In the iterative approach, we use the dynamic structure of the factor model. The monthly factor $f_t^{(m)}$ is forecast over the quarterly horizon h (i.e. over hm monthly periods) with the VAR on the factor in equation (2). The forecast of y_t is then derived from an equation that relates y_t to the contemporaneous values of the factors and their lags:

$$y_{t+h} = \beta_0(S_{t+h}) + \sum_{i=1}^{r} \beta_{1,i}(S_{t+h}) B(L^{1/m}, \theta_i) f_{i,t+h}^{(m)} + \varepsilon_{t+h}(S_{t+h}), \qquad t = 1, \ldots, T-h \quad (8)$$

[4] See Chevillon and Hendry [2005] and Marcellino, Stock and Watson [2006] for a recent discussion on this issue in single-frequency models.

In the direct approach, no forecast of the factor is made. Instead, the forecast model of y_t is specified and estimated as a linear projection of the h-step ahead variable y_t on an intercept and the estimated factors:

$$y_{t+h} = \delta_0(S_{t+h}) + \sum_{i=1}^{r} \delta_{1,i}(S_{t+h})B(L^{1/m}, \theta_i)f_{i,t}^{(m)} + \varepsilon_{t+h}(S_{t+h}), \qquad t = 1, \dots, T-h \quad (9)$$

The two equations are estimated for $t = 1, \dots, T-h$ and the forecast of y_t in $T+h$ is derived by weighting each estimated regime with the predicted probabilities of the two states in $T+h$. The forecast of the chain S_t at horizon h is given by:

$$P(S_{T+h} = 1 \mid I_T; \Theta) = (p_{11} + p_{22} - 1)^h (P(S_T = 1 \mid I_T; \Theta) - \xi_1) + \xi_1 \quad (10)$$

where the last term is the unconditional probability of state 1: $\xi_1 = \frac{1-p_{22}}{2-p_{11}-p_{22}}$.

From this, we see that the MS-FaMIDAS model introduced in this paper enables us to perform short-run GDP forecasts, while taking account of regime shifts. First, the MIDAS regression incorporates indicators sampled m times over the basic time unit. Hence, the MIDAS specification makes it possible to incorporate within-quarter information to update the GDP forecast and the probability of the state m times during the quarter in a very direct way. Moreover, this approach can be implemented when some observations are missing at the end of the sample due to publication lags by applying the Kalman filter. It can also deal with an irregular pattern in the missing observations (the so-called *ragged edge* problem) caused by different time releases of the indicators. More generally, the Kalman filter allows us to use the information provided by variables available on different sample periods.

2 Monte Carlo simulations

2.1 Experimental design

We use Monte Carlo experiments to check the robustness of the estimation procedure and to assess the performance of the MS-FaMIDAS model relative to several benchmarks, both in-sample and out-of-sample.

Our Monte Carlo experiment involves the following steps.

1. Simulation of a MS-factor MIDAS model:

a. Simulation of the high frequency factor $f_t^{(m)}$, $t = 1/m, 2/m, \ldots, T$ with $f_t^{(m)}$ a r-dimensional process following VAR(p) dynamics in which errors are normally distributed and generated via a pseudo-random number generator.

b. Construction of N observable variables $x_{it}^{(m)}$ according to $x_{it}^{(m)} = \lambda_i' f_t^{(m)} + e_{it}^{(m)}$, $t = 1/m, 2/m, \ldots, T$ where λ_i and $e_{it}^{(m)}$ are *i.i.d.* standard normal and generated via a pseudo-random number generator.[5]

c. Simulation of the low frequency variable y_t, $t = 1, 2, \ldots, T$ according to equation (6) where S_t is a simulated first-order Markov chain.

2. Estimation of the relationship between the low and high frequency variables $y_t = g(f_t^{(m)}) + \varepsilon_t$, $t = 1, \ldots, T$ with alternative specifications of $g(.)$ detailed below.

We simulate $T \times m$ observations of the high frequency indicators X and T observations of the low frequency variable Y, with m the number of times the high frequency indicators X are sampled between two time units of Y. We replicate these steps $R = 1000$ times.[6]

The set of parameters is chosen close to the empirical set-up in section 3 with $m = 3$, a sample size $T = 200$ quarters (i.e. 600 observations of the high frequency variables for $m = 3$), $r = 1$ factor driven by $q = 1$ shock and extracted from $N = 50$ monthly variables. We assume that the factor follows an AR(1) process, where the autoregressive coefficient φ is equal to -0.3 (such a value is relevant for a factor extracted from financial data). The coefficients of the MS-factor MIDAS model used when simulating the dependent variable are given below:

$$(p_{11}, p_{22}, \beta_{0,1}, \beta_{1,1}, \beta_{0,2}, \beta_{1,2}, \theta_1, \theta_2, \sigma_1, \sigma_2) = (0.95, 0.85, 0.5, -1, -0.5, 1, 2, -0.15, 0.3, 0.2)$$

In our application to US output growth rate, the shorter state 2 characterized by a lower mean and lower volatility corresponds to the recession regime. Finally, we consider two possible lag lengths $K = \{5, 12\}$.

Several specifications are estimated from the simulated observations of y_t. First, we estimate the MS-factor MIDAS model in order to assess the robustness of the estimation procedure. We also consider alternative specifications to measure the loss due to the time-aggregation of information and/or the omission of the non-linear dynamics. Formally, six models are considered. The first three specifications are linear and the last three equations are MS models (the last one is the MS-FaMIDAS specification which is the true model): where the polynomial $B(L^{1/m}, \theta)$ is defined in equation (4).

[5] A similar experimental design is used in Bai and Ng [2007].

[6] In step 1a, the first 100 simulated observations of the factors f_t and the Markov chain S_t are discarded to remove the effect of the initial conditions.

$$y_t = \beta_0 + \sum_{i=1}^{r} \sum_{j=1}^{\lceil K/m \rceil} \beta_{i,j} L^j \hat{f}_{i,t} + \varepsilon_t \quad \text{(ML1)} \qquad y_t = \beta_0(S_t) + \sum_{i=1}^{r} \sum_{j=1}^{\lceil K/m \rceil} \beta_{i,j}(S_t) L^j \hat{f}_{i,t} + \varepsilon_t(S_t) \quad \text{(MS1)}$$

$$y_t = \beta_0 + \sum_{i=1}^{r} \sum_{j=1}^{K} \beta_{i,j} L^{j/m} \hat{f}_{i,t}^{(m)} + \varepsilon_t \quad \text{(ML2)} \qquad y_t = \beta_0(S_t) + \sum_{i=1}^{r} \sum_{j=1}^{K} \beta_{i,j}(S_t) L^{j/m} \hat{f}_{i,t}^{(m)} + \varepsilon_t(S_t) \quad \text{(MS2)}$$

$$y_t = \beta_0 + \sum_{i=1}^{r} \beta_{1,i} B(L^{1/m}, \theta_i) \hat{f}_{i,t}^{(m)} + \varepsilon_t \quad \text{(ML3)} \qquad y_t = \beta_0(S_t) + \sum_{i=1}^{r} \beta_{1,i}(S_t) B(L^{1/m}, \theta_i) \hat{f}_{i,t}^{(m)} + \varepsilon_t(S_t) \quad \text{(MS3)}$$

In equations (ML1) and (MS1), the factors are converted to quarterly frequencies by averaging the months of the quarter. We choose a number of quarterly lags consistent with the true monthly lag, given by the closest quarterly lag above or equal to the monthly lag in the DGP. By comparing these two equations with the following ones, we measure the loss due to information aggregation. Equations (ML2) and (MS2) are MIDAS models with unrestricted lag polynomials; they are also considered by Marcellino and Schumacher [2010].[7] In equations (ML3) and (MS3), we use the Almon polynomial as defined in equation (4) to obtain a more parsimonious specification. The specifications (ML2) and (MS2) do not impose any structure on the coefficients of the lagged factors (such as the one implied by the exponential Almon function), but are far less parsimonious.[8]

We apply several criteria to compare the ability of the six specifications to capture the dynamics of y_t and S_t. First, we use the traditional R-squared and Bayesian information criteria. In the case of MS models, the R-squared is derived by weighting the residuals of each regime by the predicted probability $P(S_t \mid I_{t-1}; \Theta)$. To assess the quality of regime estimation, we use the quadratic probability score (QPS) given by:

$$\frac{1}{T} \sum_{t=1}^{T} (P(S_t = 1 \mid I_T; \Theta) - S_t)^2 \qquad (11)$$

with $P(S_t = 1 \mid I_T; \Theta)$ the smoothed probability of being in state one.

In order to assess the regime estimation, we will also consider a new criterion: a Turning Point Indicator (TPI hereafter). This indicator aims at assessing the model's

[7]Foroni et al. [2012] also compare the MIDAS specification based on distributed lag polynomials with the unconstrained MIDAS model with a single high frequency regressor. They study the relative performance of the two specifications on simulated data and for nowcasting euro area and US GDP. They show that the MIDAS model with unrestricted lag polynomials can outperform the restricted MIDAS model, especially for small differences in sampling frequencies (i.e. for small m). See also Barsoum and Stankiewicz [2013] for similar results in a MS-MIDAS model.

[8]For instance, for $K = 12$ and $r = 1$, we need to estimate 30 parameters in the MS unconstrained model (MS2), as opposed to only 10 parameters in the MS Almon specification (MS3).

ability of the model to detect each turning point accurately or with a lead / lag of τ quarters.

$$TPI(\lambda, \tau) = \frac{1}{n} \sum_{t=1}^{T} \max_{-\tau \leq h \leq \tau} [(P_{t-h}(\lambda) - P_{t-h-1}(\lambda))(S_t - S_{t-1})] \tag{12}$$

where n is the number of observed turning points, $P_t(\lambda) = \mathbb{1}(P(S_t = 1 \mid I_T; \Theta) > \lambda)$ with the indicator function $\mathbb{1}(\cdot)$ and the threshold parameter λ equal to 0.5 or 0.4 in our application. Compared to the QPS criterion, this index focuses on the periods with a change of regime.

2.2 In-sample evaluation

This section presents the results of the in-sample evaluation of the MS-factor MIDAS model. At this stage, the specification is estimated on the whole sample and a balanced dataset is used. The results are provided in Table 1.

First, we assess the robustness of the two-step approach to estimate the MS-FaMIDAS model. Table 1a provides the average estimates of the coefficients of the model and the standard deviations of the estimates in the 1000 replications. For parameters θ_1 and θ_2, we also provide an average measure of the error on the weights given by:

$$\frac{\sum_{j=1}^{K} [b(j, \hat{\theta}) - b(j, \theta)]^2}{\sum_{j=1}^{K} b(j, \theta)^2} \tag{13}$$

As noted by Guérin and Marcellino [2013], it is more important to correctly estimate the shape of the function rather than the point estimates of θ_1 and θ_2.

[INSERT TABLE 1 HERE]

When the DGP is correctly identified, the estimation procedure provides accurate estimates of the parameters. Indeed, the average estimates are generally very close to the parameters of the underlying DGP and dispersion is low. Note that the estimated parameters of the shortest regime display higher volatility. This is not surprising since this regime is less frequently visited. The estimated parameters of the Almon function, θ_1 and θ_2, are also less accurate, especially for small values of K, and the estimates of these two parameters show higher dispersion. However, the approximate error remains very low, even for the smallest values of K. In addition, the goodness-of-fit is high, as shown by the large R-squared. This quality decreases with the Almon lag K, which can be related to the increase in the approximation error for large K.

Second, we compare in Table 1b the six specifications (ML1-3) and (MS1-3) on the simulated data to measure the loss due to time aggregation and/or omission of the non-linear dynamics. The comparison is done for the parameters of the reference set-up. To assess the impact of changes in the reference set-up, we consider alternatively a lower persistence of the recession regime $p_{22} = 0.70$, a smaller sample size $T = 120$ quarters (i.e. 360 months), fewer variables $N = 25$, a flatter weighting function obtained for smaller values of $\theta = \{0.2, -0.015\}$ and $r = 2$ factors driven by $q = 2$ shocks. We also compare the weakly persistent AR(1) process for the factor with $\varphi = -0.3$ (suitable for factors extracted from financial data) with a more persistent process with $\varphi = 0.8$ (more appropriate for real and survey data). Finally, we consider a larger difference in sampling frequencies $m = 12$ (which corresponds to the case of a quarterly variable regressed on weekly data). We give the results of all these configurations for $K = \{5, 12\}$.[9]

Overall, we find a significant loss when omitting the regime switches and the mixed frequencies present in the simulated data. First, there is a loss when the high frequency indicators are converted to lower frequency one with simple time averaging (ML1 relative to ML2-3 and MS1 relative to MS2-3). This loss is larger for small K. The R-squared can drop by up to 40% compared to the MIDAS specification. The two regimes are also better identified in the MIDAS specification, as indicated by lower values of the QPS criterion. Moreover, there is a marked loss when the non-linear dynamics (MLi relative to MSi) is not taken into account. The R-squared can post a decrease of up to 90% in MLi relative to MSi (or a decrease of 65% in the BIC criterion penalizing the number of parameters in the MS model).

Among the MS models, the two MIDAS specifications perform similarly in terms of fit and regime identification, although MS3 is much more parsimonious than MS2. This is due to the flexibility and relative parsimony of the unconstrained MIDAS specification for small values of K. This is less true for large value of K and in the case of a larger difference between frequencies $m = 12$ (BIC criteria much higher in MS2), as already noted by Foroni et al. [2012] in a linear framework and with a single explanatory variable.

Changes in the reference parameters have an impact on the performance of the MS-

[9] When $m = 12$, we consider a higher number of lags $K = \{20, 48\}$ for X in order to condition Y on the same number of quarterly lags of X and we simulate $T \times m = 2400$ weekly observations of X.

FaMIDAS model. The relative gain of MS3 diminishes when the sample size T or the number of cross-sections N is lower and when the number of factors r increases. Moreover, the linear approximation is less detrimental when the volatile regime is shorter-lived ($p_{22} = 0.70$). In contrast, simulation results show a large improvement in the MS-FaMIDAS estimation accuracy relative to the linear model when the factor is more persistent. This result is probably due to a better estimation of the non-linear dynamics with a more persistent factor. Finally, a flatter distribution of the weights with lower values of θ leads to a smaller gain of the MIDAS approach compared to a trivial mean over the quarter (MS3 relative to MS1). For a distribution with fewer variations in the weights over the quarter, no significant gain is even found for K multiple of 3. Nevertheless, the MIDAS approach performs better when information is relevant over an incomplete quarter (e.g. for $K = 5$). Indeed, the MIDAS specification excludes the irrelevant months, while MS1 gives an equal weight to each monthly lag of the quarters including the last one(s) not present in the DGP.

2.3 Forecast evaluation

We now turn to the out-of-sample evaluation of the model forecasts at different horizons and with unbalanced datasets.

The experimental design is the following. We still generate data from a MS-factor MIDAS model, as described previously, but we remove the last observations of the simulated sample. We estimate the six specifications (ML1)-(ML3) and (MS1)-(MS3) over the rest of the period and forecast the variable y_t at horizon h with the direct or iterative approach. We recursively expand the sample and repeat these calculations up to the last quarter of the out-of-sample period. We finally get three sets of forecasts at each quarterly horizon, made at each month of the quarter for each quarterly observation of y_t in the out-of-sample period. We replicate R times the whole procedure for different forecast horizons h.

In real-time applications, the datasets typically contain missing observations for certain time series at the end of the period due to different publication lags. To address this issue, we do not use a balanced dataset. We suppose instead that the set of N monthly indicators is released with different delays of publication, ranging from 0 to 2 months. The delays that we consider are typical of those in short run forecasting of real GDP. For this purpose, the practitioner uses survey data and financial series available during the month to which they refer, while hard indicators such as retail sales or industrial production indices are released one or two months later. In the recursive scheme, we replicate the pattern of missing values at the end of each sample.

We use the reference parameters considered in the in-sample evaluation with $N = 50$ and $T = 200$ (that is 600 monthly observations). We provide the results for two alternative numbers of lags of the high frequency variables $K = 5$ (non multiple of 3) and $K = 12$. In addition, we suppose that among the 50 monthly indicators, 30 are published during the reference month, 15 one month later and 5 two months later. These proportions correspond to the composition of the sample in our empirical application where we use a majority of financial variables. The out-of-sample window contains 120 monthly observations and we consider seven forecast horizons: $h = 0, 1/3, 2/3$ (nowcasting) and $h = 1, 4/3, 5/3, 2$ (forecasting).

As in the in-sample evaluation, several criteria are applied to assess the forecast of y_t and S_t. First, we use the usual root mean squared forecast error (RMSFE) to assess the quality of the forecast of y_t. To measure the quality of the regime forecast, we also use the quadratic probability score (QPS) and the turning point indicator (TPI) defined in (11) and (12), where the smoothed probability is replaced by the Markov chain prediction in (10). In the TPI, we choose a threshold parameter λ equal to 0.5 and a lead / lag $\tau = 2$.[10]

Using these criteria, we compare the performance of the six models against two usual benchmarks: an autoregressive process of order 1 with a constant and a random walk with drift. In the case of the AR process, we use a one-period-ahead model iterated forward for the desired number of periods in the iterative approach and a regression of the h-ahead values of the variable on its current and past values in the direct approach. The forecast derived from the random walk is obtained as the average of the past values of the endogenous variable computed at every recursion.

The results obtained for $R = 200$ replications are shown in Table 2. Overall, the findings of the in-sample analysis remain valid.

[INSERT TABLE 2 HERE]

The first part of the Table shows the RMSE ratio against the random walk (a ratio below one indicates a gain relative to the naive benchmark). Among the eight specifica-

[10] The results are qualitatively similar for other values of these two parameters. They are not reported here for the sake of parsimony.

tions, MS3 provides the best quantitative forecasts. The model does not outperform the random walk for large horizons, but the quality of forecasts gradually increases as the horizon shortens and more information is available on the quarter to be forecast. The aggregation of information degrades forecast accuracy (ML1 versus ML2-3 and MS1 versus MS2-3). The difference is sharper for $K = 5$, which is not a multiple of 3, as found in the previous section. Similarly, the omission of the non-linearity worsens the criteria (ML relative to MS). Among the MS models, the performance of MS2 is fairly close to that of MS3, even for $K = 12$, despite the proliferation of parameters for large values of K.

Regarding the forecast of the chain (QPS and TPI criteria), the results are also supportive of the mixed frequency models (MS2 and MS3). The QPS criterion is smaller in the MS2 and MS3 specifications and the proportion of identified turning points is higher. Once again, the difference is more pronounced for $K = 5$. When $K = 12$, the QPS criteria are similar in the three models, but the MS3 specification outperforms the MS1 model according to the TPI criterion for short horizons. The performance of MS2 and MS3 is similar according to the two criteria.

Finally, we do not find any strong differences between the iterative and direct approaches. The direct approach provides more accurate forecasts of y_t and S_t at shorter horizons for $K = 5$ (except for $h = 0$, where the results are identical by construction). By contrast, the iterative approach performs better for large h. For $K = 12$, the results are also more supportive of the direct approach. The iterative approach performs slightly better only for $h = 2$. Overall, the MS-faMIDAS model appears particularly relevant for nowcasting.

2.4 Effect of a potential omission of MS in the factor model

The existing literature provides theoretical and empirical evidence showing that the results of factor-based regressions are robust to a limited continuous time variation or a few limited discrete jumps in the loadings and in the persistence of the factors (see Banerjee et al. [2008], Stock and Watson [2009], Bates et al. [2013]). In this final part of section 2, we assess via Monte Carlo simulations in finite sample, the effect of Markov-switching in the factor representation.

To simulate data, we extend the experimental design as it is defined in subsection 2.1. The low frequency variable y_t is still simulated according to the MS-FaMIDAS equation

(MS3) but we introduce regime switches in the factor model. We consider two possible cases. First, we allow changes in the AR equation of the factor:

$$f_t^{(m)} = c_{S_t^{(m)}} + \varphi_{S_t^{(m)}} f_{t-1}^{(m)} + \sigma_{S_t^{(m)}} u_t^{(m)} \tag{14}$$

where $S_t^{(m)}$ is the first-order Markov chain in the equation of y_t converted to the monthly frequency (for simplicity, we suppose that the regime is the same in the three months of the quarter). In the first regime, we set $(c_1, \varphi_1, \sigma_1) = (2, -0.3, 1.0)$, while the parameters of the second regime are given by $(c_2, \varphi_2, \sigma_2) = (1.5, 0.1, 0.5)$. Finally $u_t^{(m)}$ is i.i.d. N(0,1).

Second, we consider the effect of regime switches in the loadings:

$$x_{it}^{(m)} = \lambda'_{iS_t^{(m)}} f_t^{(m)} + e_{it}^{(m)} \tag{15}$$

where λ_{i1} and $e_{it}^{(m)}$ are obtained from independent draws of a standard normal distribution. In the second state, the loadings are given by $\lambda_{i2} = \lambda_{i1} + \delta$. In the simulation design, the parameter δ is set to 1. A similar process with one single break is considered in Breitung and Eickmeier [2011].

The three specifications (MS1)-(MS3), which omit the changes in the factor representation, are then estimated on these simulated data. We still conduct 1000 Monte Carlo replications to compute the average R-squared and QPS criteria in the in-sample framework and 200 replications for the computation of the average RMSFE relative to the random walk benchmark and QPS criteria in the out-of-sample assessment. The results are reported in Table 3. Each table compares the criteria with the ones obtained without regime changes in the factor model. A ratio superior to one indicates a loss relative to the reference case. Overall, ignoring the presence of MS in the factor model does not lead to a marked deterioration of the estimation and forecast of y_t and S_t.

[INSERT TABLE 3 HERE]

In the in-sample evaluation, the R-squared of the MS-factor model MS1 and the MS-factor MIDAS models MS2 and MS3 are relatively unchanged when we allow a variation in the loadings or in the persistence of the factor (the ratios are very close to one in both cases). The regime identification tends to be more affected, as shown by the increase in the QPS coefficient by 5% in MS3 in the case of regime-switches in the loading structure of the factor and by 10% to 15% in MS3 with a variation in the persistence of the factor. The QPS criterion posts an increase up to 22% in the unconstrained MS-FaMIDAS model MS2. However, even in this last case, the QPS value is still very low (around 0.02).

The findings are similar in the out-of-sample evaluation.[11] In the case of MS in the factor loadings, there appears to be only a slight deterioration of the forecast accuracy of y_t with a loss below 3%. The forecast of y_t appears unaffected too by the introduction of MS in the AR dynamics (we even find a small improvement which may be due to a global lower volatility of the process[12]). However, the forecast of the chain worsens in both cases, especially in the case of MS in the factor dynamics for $K = 5$, but the loss is not very large. The maximum increase of the QPS criterion amounts 14% but the loss is generally below 10%.

Overall, the introduction of regime switches in the factor representation has a rather limited impact on the estimation and forecast of y_t and S_t for reasonable changes in the parameters of the factor model. This result is reassuring, even if it may not hold for all simulation designs.[13] A more extensive evaluation would be necessary but it is beyond the scope of the paper.

3 Application to US GDP

3.1 The data

In this section, we apply the MS-FaMIDAS to model the relationship between US GDP and a set of higher frequency financial variables.

The sample period is 1959Q1-2010Q4 which includes eight recessions (identified by the National Bureau of Economic Research). The database consists of US real output growth rate (quarterly) and the block of financial variables (monthly) considered by Stock and Watson [2005].[14] Our dataset was collected in September 2011. We use vintages of output

[11]For the sake of parsimony, the results of the iterative approach are not reported since they are very similar.

[12]The variance in the first state is equal to the variance in the reference setting with no MS in the factor representation and the variance is lower in the second state.

[13]There should be a detrimental effect on the rank estimation too and thus on the estimation of the number of the factors (see Breitung and Eickmeier [2011], Bates et al. [2013] in the case of a single break in the loadings of the factor), but this question is not studied here.

[14]With the exception of the M3 monetary aggregate and the index of sensitive materials price since these variables are only available up to 2004Q2.

growth from the real-time datasets constructed by Croushore and Stark [2001] (see also Croushore [2011] for a more recent survey on the use of real time data). The financial variables (except NAPM) are extracted from Datastream. The database includes money and credit quantity aggregates, stock prices, interest rates and spreads, exchange rates and price indices. The 56 variables are listed in Appendix 1, together with their source and their transformation.

Many financial variables (e.g. stock prices and price indices) are also available at a daily frequency. Using financial daily information can improve quarterly forecast of GDP, as shown by Andreou et al. [2013] in a MIDAS framework. However, the daily variables are available on a shorter span (from 1999 in Andreou et al. [2013] for most of them). Yet, as we are interested in detecting recessions, we need a longer time period for this purpose. Moreover, money and credit aggregates released on a monthly basis should be considered separately. For this reason, we follow Guérin and Marcellino [2013] in our application and we consider financial indicators only at a monthly frequency.

All series have been transformed to stationarity by using logarithms, differences, or log-differences. In addition, all series are standardized to mean zero and variance one. US quarterly GDP data are released about one month after the end of the reference quarter. The financial variables are released during the month to which they refer, while the monetary aggregates and the price indices have a publication lag of one month (in the out-of-sample evaluation, we make the assumption that the publication lags do not change over time).[15] In the out-of-sample evaluation of the model, we compare the GDP forecasts with the final values of output growth approximated by the final vintage available in September 2011. We assume that the financial variables are not revised.

3.2 Estimation results

We first provide the estimation results of the MS-factor MIDAS model on US data over the last five decades.

There are a number of parameters to be specified related to the factor model. We consider $r = 1$ factor driven by $q = 1$ dynamic shock.[16] The order p of the AR on the

[15] At this level, the 2-step method of Doz et al. [2011] is useful in forecasting to exploit this information in an optimal way. In the first step of the method, the factors are extracted on the range of observations, excluding the last observations of the set of indicators published without any delay. In the second step, they are re-estimated with the Kalman filter on the whole range of observations.

[16] At this level, we choose a smaller number of factors than the one implied by the criteria of Bai and

factor is chosen with the usual BIC criterion in the range {1,2,3} and is set to 3. We retain $K = 12$ lags so that the specification includes the last year of monthly data. This choice is supported by the estimated profile of the weight function (see below). Finally, as suggested by Psaradakis, Sola, Spagnolo and Spagnolo [2009], we rely on the AIC criterion to select three regimes in the MS models.

The estimation results of the MS-FaMIDAS model with three regimes are given in Table 4a. The link between GDP dynamics and the financial factor is found significant, except in the high growth state. Guérin and Marcellino [2013] obtain similar results when estimating the link between US GDP and the yield curve or the S&P500 index. The difference in the regime intercepts is large, negative during the recession state and positive during the two expansion states. We also find the classical features of the business cycle: the expansionary state is longer-lived and more volatile. Figure 1 shows the smoothed probabilities of being in recession according to the MS-FaMIDAS model.[17] The grey areas represent NBER recession periods. The results are quite favorable to the new specification. The model successfully detects the eight recessions over the period 1959-2010.

[INSERT TABLE 4 AND FIGURE 1 HERE]

Figure 2 depicts the estimated weight function. We find a hump-shaped profile. The estimated function flattens around $K = 12$. According to this chart, the financial factor contains useful information for predicting the US business cycle up to 12 months ahead. This result is consistent with the literature on the predictive content of financial variables (See Stock and Watson [2003] for a survey). The weighting function peaks at the fifth month, which means that the financial factor contains particularly relevant information on the business cycle at this horizon. It should also be noted that the function is sharp. This is a favorable configuration for the MIDAS specification according to the results of the Monte Carlo simulations reported in section 2.

[INSERT FIGURE 2 HERE]

Ng [2002] and Bai and Ng [2007]. Relying on these criteria, we should consider a model with $r = 4$ factors (without any lag in the equation of x_t). However, the large number of parameters (32) contained in this model penalizes the specification both in-sample and out-of-sample. The model with 4 factors fits the US data more accurately but the regime identification deteriorates markedly with a much higher QPS value and a drop in the percentage of detected turning points. The model performs also badly to forecast the US GDP as well as to detect recessions. The results are available from the authors upon request.

[17] See Appendix 2 for a plot of the smoothed probabilities in the three regimes.

In Table 4b, we compare the goodness-of-fit of the six competing models (ML1-ML3) and (MS1-MS3), as well as several benchmark models: a simple AR(1), a MS model with an intercept only and a switch on the intercept and the variance (MSIH) and an AR(1) process with a switch on all coefficients (MSIARH). The results reported in Andreou et al. [2013] suggest that adding an autoregressive term in the specification ML3 can be useful (FADL-MIDAS). This class of models is considered as an additional linear benchmark.

The results show that the MS-FaMIDAS models (MS2 and MS3) fit the US data accurately and better identify the states of the economy. Incorporating regime switching coefficients enhances the fit of the models (MS versus ML models). Yet, the more parsimonious FADL-MIDAS model displays the lowest BIC. Including factors extracted from financial data also improves the fit of the models, as shown by the comparison of FADL-MIDAS with the autoregressive benchmark and the comparison of (MS1)-(MS3) with MSIH and MSIARH. The financial factor also enhances the detection of recessions in the non-linear specifications. A sizeable gain is achieved when using higher frequency data (MS2 and MS3 versus MS1). The QPS criterion is smaller in MS2 and MS3 and the turning points are much better identified when incorporating mixed frequencies according to the TPI criteria, especially in the MS-MIDAS model with unrestricted lag polynomials MS2.

3.3 Out-of-sample results

Finally, we assess the quality of forecasts made seven to one month prior to the release of GDP data.

The evaluation is conducted in real time conditions. First, the models are estimated from the observations available at the time of the forecast. At this level, we use the vintages of output growth provided by Croushore and Stark (financial variables are supposed not to be subject to data revisions). Moreover, we estimate the parameters of the models recursively using the only information available at the time of the forecast. The models are estimated from 1959 and the out-of-sample period spans from 1990Q1 to 2010Q4, which includes three recessions. Rather than using a balanced dataset, we also replicate the pattern of missing values at the end of the sample to take account of the time of publication of the variables, given in Appendix 1. The missing values at the end of the sample are interpolated using the Kalman filter, as explained in section 1.

More precisely, the approach is as follows. The first quarter of 1990 is forecast conditional on the information available in September 1989, in October 1989, and so on up to March 1990 (the US quarterly GDP is released about one month after the end of the reference quarter, e.g. GDP in 1990Q1 is released in April 1990). The MS-FaMIDAS model is estimated on these data-sets and is used for prediction. Similarly, we produce seven forecasts of the GDP growth rate in the second quarter of 1990 from the data available in December 1990 to June 1990. These calculations are replicated up to the last quarter of the out-of-sample period. We finally obtain seven sets of forecasts of the GDP growth rate and of the occurrence of the recessionary state for the quarters 1990Q1 to 2010Q4. The GDP forecasts are compared to the final value of GDP approximated by the series available in September 2011 and the chain forecast is compared to the NBER business cycle dates.

Table 5 shows the ratios of the RMSFEs of the competing specifications against the RW benchmark, as well as the QPS and TPI criteria for the MS models. We also distinguish the results obtained in the direct and iterative approaches.

[INSERT TABLE 5 HERE]

Among the 12 specifications, the MS-FaMIDAS model MS3 provides good quantitative forecasts (RMSE criterion). First, MS3 dominates clearly the linear models: the basic AR(1) model, as-well as models including financial factors. Among the MS models, MS3 almost always yields lower RMSE than MSIH and MSIARH. This result shows that the financial factor provides useful information for forecasting US GDP growth. Moreover, the parsimonious MS3 specification performs much better than the MS-MIDAS with unrestricted lag polynomials MS2, almost at all horizons. This contrasts with the results of the in-sample evaluation and of the Monte Carlo simulations. This may be due to the large number of parameters to estimate in a three state model.

Regarding the detection of recessions, the results are also supportive of the MS-FaMIDAS specification MS3. The smallest values for the QPS criterion are obtained with this model, in particular with the iterative approach. MS3 also detects a higher proportion of the observed turning points at the shortest horizons according to the TPI criterion. However, we note at this level that MSIH and MS1 compare favorably with the more sophisticated specifications. By contrast, the unconstrained MIDAS specification MS2 performs very badly around the turning points. Again, the large number of parameters in a 3-state model penalizes this specification in the out-of-sample evaluation.

Finally, we note again that there is no clear difference between the iterative and direct approaches. This result is in line with the findings emerging from the Monte Carlo simulations. One possible explanation for this result is the short forecasting horizons considered in the paper.

4 Concluding remarks

In this paper, we introduce the MS-factor MIDAS model. This specification deals with several issues specific to short-run forecasting. It enables us to exploit the information provided by a large data-set consisting of mixed frequency variables. In addition, it yields quantitative and qualitative information about the state of the economy.

Monte Carlo evidence suggests that our estimation procedure provides robust estimates of the parameters of the model. The Monte Carlo experiments also show that the MS-FaMIDAS model represents a robust forecasting device for various settings. We find a significant loss, both in-sample and out-of-sample, when omitting the regime switches and the mixed frequencies present in the simulated data. In line with previous research on MIDAS models, we also establish that the MIDAS specification with unrestricted lag polynomials performs equally to the MIDAS models with constrained lags, at least when the difference in sampling frequencies is small. These results are found robust to the presence of regime switches in the factor representation.

In the empirical application, we show that the MS-FaMIDAS model provides a better fit of the US GDP growth rate than linear specifications when using the information provided by monthly financial indicators over 1959-2010. The MS-FaMIDAS also detects the eight recessions more successfully than the specifications that time aggregate the high frequency indicators. The findings are also supportive of the MS-FaMIDAS model in the case of out-of-sample results. The quality of forecasts of the US GDP growth rate is better than the ones of usual benchmarks. Moreover, the MS-FaMIDAS model provides more accurate forecasts of the recessionary state.

There are a number of potential extensions to this paper. In particular, it would be interesting to allow a switch in the parameters of the weighting function. Moreover, it would be helpful to incorporate in the model the information provided by monthly and quarterly macroeconomic indicators as done in Andreou et al. [2013]. Considering how daily financial data could improve our empirical results is also on our research agenda.

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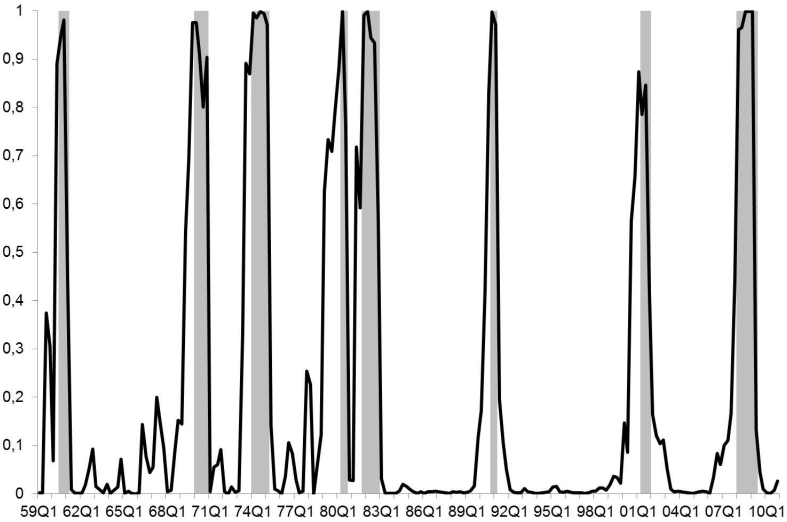
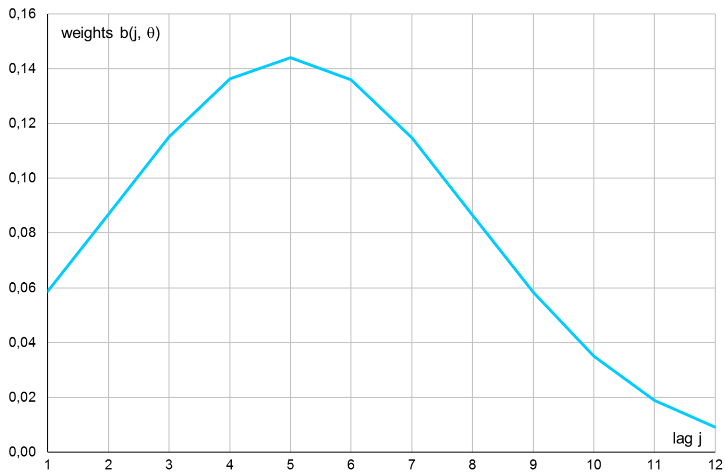
Figure 1: Smoothed probabilities of being in recession



Notes: This graph plots the US smoothed recession probabilities of the MS-FaMIDAS model (MS3). The shaded areas represent the NBER recession periods.

Figure 2: Estimated weights of the MS-FaMIDAS model



Notes: This graph depicts the estimated weight function of the MS-FaMIDAS model (MS3) on US data.

Table 1: In-sample evaluation with Monte Carlo simulations

A. Estimate accuracy of the MS-FaMIDAS model on simulated data

K	$\varphi=-0.3$	$p_{11}=0.95$	$p_{22}=0.85$	$\beta_{0,1}=0.5$	$\beta_{1,1}=-1$	$\beta_{0,2}=-0.5$	$\beta_{1,2}=1$	$\theta_1=2$	$\theta_2=-0.15$	$\sigma_1=0.3$	$\sigma_2=0.2$	apx err	R^2
5	-0.289	0.949	0.829	0.500	-0.993	-0.496	0.994	2.135	-0.165	0.309	0.209	0.004	0.86
	(0.038)	(0.021)	(0.075)	(0.028)	(0.066)	(0.092)	(0.117)	(0.92)	(0.105)	(0.019)	(0.028)		
12	-0.290	0.949	0.828	0.501	-0.990	-0.502	0.995	2.057	-0.154	0.301	0.198	0.015	0.78
	(0.039)	(0.020)	(0.069)	(0.028)	(0.108)	(0.043)	(0.152)	(0.402)	(0.030)	(0.019)	(0.027)		

Notes: Table A provides the average over the 1000 replications of the parameter estimates of the MS-FaMIDAS model (MS3) and in brackets the standard deviation of the estimates over the replications. The last two columns show the average approximation error of the weights and the average R-squared.

B. Performance of the six specifications on simulated data

		R^2						BIC						QPS			apx err	
DGP	K	ML1	ML2	ML3	MS1	MS2	MS3	ML1	ML2	ML3	MS1	MS2	MS3	MS1	MS2	MS3	ML3	MS3
benchmark	5	0.06	0.19	0.18	0.57	**0.86**	**0.86**	451.26	435.42	427.03	418.41	219.38	**193.85**	0.047	0.016	**0.016**	0.154	**0.004**
	12	0.08	0.14	0.10	0.75	**0.79**	**0.78**	357.80	388.48	349.59	236.13	270.17	**181.95**	0.020	0.020	**0.017**	0.396	**0.015**
$p_{22}=0.70$	5	0.11	0.33	0.32	0.50	**0.83**	**0.83**	420.44	375.80	367.34	410.06	230.23	**205.71**	0.057	0.021	**0.019**	0.031	**0.004**
	12	0.15	0.21	0.17	0.68	**0.73**	**0.72**	309.22	336.38	297.56	241.04	271.12	**191.73**	0.025	0.026	**0.020**	0.132	**0.014**
$T=120$	5	0.07	0.22	0.20	0.56	**0.86**	**0.86**	271.40	262.21	254.83	257.51	142.14	**126.85**	0.056	0.019	**0.017**	0.230	**0.007**
	12	0.10	0.18	0.12	0.74	**0.79**	**0.78**	216.45	244.02	209.37	154.74	181.71	**118.55**	0.024	0.024	**0.018**	0.605	**0.026**
$N=25$	5	0.06	0.18	0.17	0.56	**0.85**	**0.84**	451.66	437.65	429.30	421.05	240.74	**215.62**	0.050	0.019	**0.018**	0.128	**0.005**
	12	0.08	0.13	0.09	0.74	**0.79**	**0.78**	358.86	389.81	350.79	241.01	276.71	**189.07**	0.021	0.020	**0.017**	0.402	**0.016**
$\theta_1=0.2$	5	0.08	0.13	0.12	0.72	**0.80**	**0.80**	367.97	373.32	364.99	258.26	207.50	**183.34**	0.024	0.018	**0.017**	0.281	**0.004**
$\theta_2=-0.015$	12	0.06	0.10	0.06	0.76	**0.78**	**0.76**	339.19	373.07	333.61	199.90	260.75	**179.20**	0.017	0.020	**0.016**	0.847	**0.016**
$r=2$	5	0.23	0.28	0.24	0.82	**0.87**	**0.88**	526.81	545.15	532.96	355.47	344.72	**281.30**	0.028	0.021	**0.018**	0.464	**0.166**
	12	0.23	0.31	0.22	0.81	**0.85**	**0.87**	535.98	599.53	526.67	388.44	509.84	**280.72**	0.031	0.027	**0.021**	0.527	**0.125**
$\varphi=0.8$	5	0.23	0.25	0.24	0.90	**0.93**	**0.93**	542.11	552.04	543.82	265.25	223.55	**197.73**	0.017	0.013	**0.012**	0.294	**0.006**
	12	0.23	0.27	0.24	0.91	**0.92**	**0.92**	514.30	547.24	508.30	223.96	271.80	**182.10**	0.018	0.016	**0.014**	0.648	**0.006**
$m=12$	20	0.05	0.17	0.10	0.68	**0.79**	**0.78**	353.94	422.12	348.60	268.87	356.04	**183.44**	0.019	0.016	**0.014**	0.386	**0.013**
	48	0.06	0.30	0.09	0.69	**0.79**	**0.78**	365.26	545.60	351.72	289.89	658.50	**182.91**	0.020	0.015	**0.014**	0.427	**0.012**

Notes: Table B reports for (ML1-3) and (MS1-3) the average R-squared and BIC criteria over the 1000 replications. The average QPS criterion is provided for (MS1-3). The last two columns show the average approximation error in the weights for the two MIDAS specifications ML1-MS1. For each criterion, entries in bold indicate the best performing model. The results are reported for the benchmark set-up $(p_{11}, p_{22}, \beta_{0,1}, \beta_{1,1}, \beta_{0,2}, \beta_{1,2}, \theta_1, \theta_2, \sigma_1, \sigma_2)$=(0.95,0.85,0.5,-1, -0.5,1,2,-0.15,0.3,0.2) with $m=3$ and for various departures from this set-up.

Table 2: Out-of-sample evaluation with Monte Carlo simulations

A. Performance of the six specifications on simulated data with $K = 5$

		Direct approach							Iterative approach						
h		2	5/3	4/3	1	2/3	1/3	0	2	5/3	4/3	1	2/3	1/3	0
	AR	**0.99**	**0.98**	0.98	0.98	0.96	0.96	0.96	1.00	0.99	0.99	0.99	0.96	0.96	0.96
R	ML1	1.01	1.15	1.00	0.99	0.99	0.99	0.99	1.01	1.03	1.00	0.99	0.99	0.99	0.99
M	ML2	1.03	1.01	0.96	0.93	0.92	0.93	0.92	1.00	1.00	0.95	0.92	0.93	0.93	0.92
S	ML3	1.01	1.01	0.97	0.93	0.92	0.92	0.92	1.00	1.00	0.95	0.92	0.96	0.92	0.92
E	MS1	1.00	1.20	0.95	0.93	0.89	0.89	0.89	0.99	1.00	0.95	0.92	0.88	0.88	0.89
	MS2	1.01	1.00	0.97	0.92	0.76	0.69	0.68	0.97	0.97	0.83	0.79	0.85	0.69	0.68
	MS3	1.00	0.99	**0.85**	**0.80**	**0.69**	**0.69**	**0.68**	**0.97**	**0.95**	**0.83**	**0.79**	**0.79**	**0.69**	**0.68**
Q	MS1	**0.18**	**0.15**	0.15	0.15	0.12	0.12	0.12	0.17	**0.15**	0.15	0.15	**0.12**	0.12	0.12
P	MS2	0.21	0.19	0.19	0.17	0.11	0.09	0.09	**0.15**	0.17	**0.13**	**0.13**	0.14	**0.09**	**0.09**
S	MS3	0.21	0.20	**0.14**	**0.13**	**0.09**	**0.09**	**0.09**	**0.15**	**0.15**	**0.13**	**0.13**	**0.12**	**0.09**	**0.09**
T	MS1	0.17	0.35	0.34	0.35	0.60	0.60	0.60	**0.08**	0.34	0.34	0.34	0.60	0.60	0.60
P	MS2	0.20	0.40	0.40	0.49	0.75	**0.83**	0.83	0.05	0.46	**0.54**	**0.59**	0.61	0.82	**0.83**
I	MS3	**0.26**	**0.42**	**0.56**	**0.54**	**0.76**	0.82	**0.80**	0.05	**0.59**	**0.54**	**0.59**	**0.70**	**0.82**	0.80

B. Performance of the six specifications on simulated data with $K = 12$

		Direct approach							Iterative approach						
h		2	5/3	4/3	1	2/3	1/3	0	2	5/3	4/3	1	2/3	1/3	0
	AR	0.97	0.94	0.94	0.94	0.85	0.85	0.85	0.97	0.94	0.94	0.94	0.85	0.85	0.85
R	ML1	1.00	1.02	0.98	0.98	0.98	0.98	0.98	1.00	0.98	0.98	0.98	0.98	0.98	0.98
M	ML2	1.03	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.98	0.98	1.00
S	ML3	1.00	0.98	0.98	0.98	0.97	0.97	0.97	1.00	0.98	0.98	0.97	0.98	0.98	0.97
E	MS1	**0.95**	0.93	0.90	0.88	0.78	0.76	0.76	**0.93**	**0.88**	**0.86**	0.86	**0.76**	**0.76**	0.76
	MS2	**0.98**	0.93	0.92	0.90	0.76	0.76	0.76	**0.93**	0.90	**0.86**	**0.85**	0.80	**0.76**	0.76
	MS3	**0.95**	**0.86**	**0.85**	**0.85**	**0.73**	**0.73**	**0.73**	**0.93**	0.90	0.88	**0.85**	0.78	0.78	**0.73**
Q	MS1	0.16	**0.13**	**0.13**	0.13	0.09	0.09	0.09	0.15	0.13	0.13	0.13	**0.09**	**0.09**	0.09
P	MS2	0.16	0.14	0.14	0.13	0.09	0.09	0.09	0.15	0.13	0.13	0.13	0.10	**0.09**	0.09
S	MS3	0.16	**0.13**	**0.13**	0.13	0.09	0.09	0.09	0.15	0.13	0.13	0.13	0.10	0.10	0.09
T	MS1	0.08	0.52	0.54	0.52	0.69	0.68	0.70	0.04	**0.49**	0.51	0.49	0.67	0.68	0.70
P	MS2	**0.12**	0.41	**0.56**	0.51	0.68	0.68	0.69	0.06	0.48	0.51	0.52	**0.68**	0.68	0.69
I	MS3	0.04	**0.52**	0.52	**0.56**	**0.72**	**0.71**	**0.71**	**0.07**	0.46	**0.52**	**0.52**	0.66	**0.70**	**0.71**

Notes: This table shows the average over the 200 replications of the RMSE relative to the RMSE of the RW forecast (a ratio below one indicates a gain relative to the naive forecast), the average QPS and TPI criteria. The TPI criterion is given for $\lambda = 0.5$ and $\tau = 2$. Bold entries indicate the best performing model according to each criterion and for each horizon.

Table 3: Impact of omitted regime switches in the factor model

A. In-sample evaluation on simulated data

	MS in AR dynamics						MS in factor loadings					
	K=5			K=12			K=5			K=12		
	MS1	MS2	MS3	MS1	MS2	MS3	MS1	MS2	MS3	MS1	MS2	MS3
R2	1.07	1.00	1.00	1.01	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00
QPS	0.95	1.22	1.15	1.10	1.18	1.10	1.02	1.06	1.05	1.05	1.04	1.05

Notes: The entries report the average R-squared and QPS criteria relative to the ones obtained without switches in the factor model. A ratio above 1 indicates a lower (superior) R2 (QPS) in the case with MS.

B1. Out-of-sample evaluation on simulated data - MS in factor dynamics

		K=5							K=12						
	h	2	5/3	4/3	1	2/3	1/3	0	2	5/3	4/3	1	2/3	1/3	0
RMSFE	MS1	1.00	1.04	0.99	0.98	0.99	0.99	0.99	1.00	0.99	0.98	0.99	1.00	1.00	1.00
	MS2	1.00	1.00	0.96	0.92	0.95	0.98	0.98	1.01	1.01	0.99	0.98	1.00	1.00	1.00
	MS3	1.00	0.99	0.97	0.95	0.98	0.98	0.98	0.99	0.98	0.98	0.98	0.99	0.99	0.99
QPS	MS1	1.10	1.08	1.08	1.09	1.05	1.05	1.05	1.09	1.08	1.09	1.09	1.09	1.08	1.08
	MS2	1.01	1.02	1.10	1.11	1.03	1.13	1.13	1.09	1.10	1.09	1.07	1.09	1.11	1.11
	MS3	0.96	0.99	1.06	1.12	1.14	1.13	1.11	1.07	1.07	1.09	1.08	1.09	1.09	1.09

Notes: The entries report the ratio of RMSE to the RW benchmark and the QPS criteria. The two criteria are given relative to the ones obtained without switches in the factor model. A ratio above 1 indicates a superior RMSFE (QPS) in the case with MS.

B2. Out-of-sample evaluation on simulated data - MS in factor loadings

		K=5							K=12						
	h	2	5/3	4/3	1	2/3	1/3	0	2	5/3	4/3	1	2/3	1/3	0
RMSFE	MS1	1.00	1.01	1.02	1.01	1.02	1.02	1.02	1.01	1.00	1.00	1.01	1.01	1.01	1.01
	MS2	1.00	1.00	1.00	1.02	1.02	1.03	1.02	1.01	1.01	1.01	1.00	1.01	1.02	1.01
	MS3	1.00	1.00	1.01	1.03	1.02	1.03	1.02	1.01	1.00	1.00	1.00	1.01	1.01	1.01
QPS	MS1	1.12	1.06	1.05	1.06	1.06	1.06	1.06	1.07	1.07	1.07	1.07	1.07	1.07	1.07
	MS2	1.07	1.10	1.12	1.10	1.02	1.10	1.09	1.05	1.07	1.08	1.07	1.07	1.09	1.07
	MS3	1.04	1.05	1.08	1.07	1.07	1.06	1.08	1.07	1.08	1.08	1.08	1.07	1.07	1.07

Notes: See Table B1.

Table 4: In-sample evaluation on US data (1959Q1-2010Q4)

A. Estimation results of the MS-FaMIDAS on US GDP data

$\beta_{0,1}$	$\beta_{1,1}$	$\beta_{0,2}$	$\beta_{1,2}$	$\beta_{0,3}$	$\beta_{1,3}$	θ_1	θ_2	σ_1	σ_2	σ_3
-0.12	-0.93***	0.80***	0.29*	1.38***	-0.65	0.56	0.06	0.74***	0.38***	0.81***
(0.175)	(0.395)	(0.050)	(0.157)	(0.161)	(0.728)	(0.382)	(0.036)	(0.105)	(0.036)	(0.087)

Notes: This table shows the parameter estimations of the MS-FaMIDAS model estimated over 1959Q1-2010Q4 and the associated standard errors in square brackets. Significance levels: *** if the coefficient is significant at 1%, ** at 5%, * at 10%.

B. Relative performance of the specifications on US GDP data

	k	R2	AIC	BIC	QPS	TPI (0.5)	TPI (0.4)
AR(1)	2	0.11	523.6	530.3	-	-	-
ML1	5	0.05	543.1	559.7	-	-	-
ML2	13	0.07	555.5	598.9	-	-	-
ML3	4	0.06	539.7	553.0	-	-	-
FADL-MIDAS	5	0.23	500.3	**517.0**	-	-	-
MSIH	12	0.51	496.6	536.6	0.126	63%	44%
MSIARH	15	0.31	495.2	545.2	0.193	6%	6%
MS1	24	0.66	498.4	578.5	0.137	63%	56%
MS2	48	**0.67**	544.3	704.5	0.075	**81%**	**75%**
MS3	17	0.64	494.2	550.9	**0.063**	69%	63%

Notes: This table shows the R-squared, BIC, QPS and TPI criteria for the linear and non-linear specifications estimated over the full 1959Q1-2010Q4 sample. Column k gives the number of parameters estimated in each model. The TPI is computed alternatively with $\lambda = 0.5$ and $\lambda = 0.4$ and with a lead/lag parameter τ equal to two quarters.

Table 5: Out-of-sample performance on US data

		Direct approach							Iterative approach						
	h	2	5/3	4/3	1	2/3	1/3	0	2	5/3	4/3	1	2/3	1/3	0
RMSE	AR	1.00	0.99	0.99	0.99	0.92	0.92	0.92	1.00	0.99	0.99	0.99	0.92	0.92	0.92
	ML1	1.12	1.10	1.09	1.09	1.09	1.15	1.04	1.10	1.10	1.10	1.09	1.11	1.10	1.04
	ML2	1.15	1.15	1.19	1.12	1.18	1.23	1.10	1.09	1.11	1.13	1.09	1.16	1.16	1.10
	ML3	1.10	1.13	1.09	1.05	1.05	1.12	0.97	1.00	1.01	1.00	1.00	1.02	0.97	0.97
	FADL-MIDAS	1.10	1.01	1.02	1.01	1.02	1.04	0.99	1.06	1.00	1.03	0.98	0.98	0.98	0.99
	MSIH	1.03	1.01	1.01	1.00	0.95	0.94	0.94	1.03	1.01	1.01	1.00	0.95	0.94	0.94
	MSIARH	1.00	0.99	0.99	0.99	0.95	0.96	0.96	1.00	0.99	0.99	0.99	0.95	0.96	0.96
	MS1	1.02	0.98	0.99	1.00	0.95	1.04	0.98	1.05	1.01	1.07	1.02	0.95	1.04	0.98
	MS2	1.00	1.04	1.02	0.95	0.97	**0.88**	0.87	1.02	1.02	1.01	0.95	0.93	**0.88**	0.87
	MS3	1.02	0.99	0.98	**0.93**	0.95	0.92	**0.81**	1.01	0.99	**0.97**	0.97	0.96	0.90	**0.81**
QPS	MSIH	0.19	0.17	0.16	0.17	0.14	0.14	0.13	0.19	0.17	0.16	0.17	0.14	0.14	0.13
	MSIARH	**0.15**	0.15	0.16	0.15	**0.13**	0.15	0.16	0.15	0.15	0.16	0.15	0.13	0.15	0.16
	MS1	0.18	0.18	0.18	0.18	0.14	0.16	0.14	0.23	0.22	0.21	0.19	0.14	0.16	0.14
	MS2	**0.15**	**0.13**	**0.13**	**0.13**	**0.13**	0.13	0.12	**0.13**	0.13	**0.13**	**0.13**	0.13	0.13	0.12
	MS3	0.18	0.16	**0.13**	**0.13**	**0.13**	0.12	**0.09**	0.17	**0.12**	**0.13**	**0.13**	**0.12**	**0.09**	**0.09**
TPI (0.5)	MSIH	0.50	0.50	0.50	0.50	0.67	**0.83**	**0.83**	**0.50**	0.50	0.50	0.50	**0.67**	**0.83**	**0.83**
	MSIARH	0.17	0.33	0.33	0.17	0.33	0.33	0.33	0.17	0.33	0.33	0.17	0.33	0.33	0.33
	MS1	**0.67**	**0.67**	**0.67**	**0.67**	0.67	0.67	0.67	**0.50**	**0.67**	**0.67**	0.50	**0.67**	0.67	0.67
	MS2	0.17	0.00	0.00	0.00	0.17	0.17	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.17
	MS3	**0.67**	0.50	0.17	0.50	**1.00**	**0.83**	**0.83**	0.33	0.17	0.33	**0.67**	0.50	**0.83**	**0.83**
TPI (0.4)	MSIH	0.50	**0.67**	**0.67**	**0.67**	0.83	**0.83**	**0.83**	0.50	**0.67**	**0.67**	**0.67**	**0.83**	**0.83**	**0.83**
	MSIARH	0.17	0.33	0.33	0.17	0.33	0.33	0.33	0.17	0.33	0.33	0.17	0.33	0.33	0.33
	MS1	**0.67**	**0.67**	**0.67**	**0.67**	0.83	**0.83**	0.67	**0.67**	**0.67**	**0.67**	**0.67**	**0.83**	**0.83**	0.67
	MS2	0.17	0.00	0.00	0.00	0.17	0.17	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.17
	MS3	**0.67**	**0.67**	0.17	**0.67**	**1.00**	**0.83**	**0.83**	0.33	0.17	0.33	**0.67**	0.67	**0.83**	**0.83**

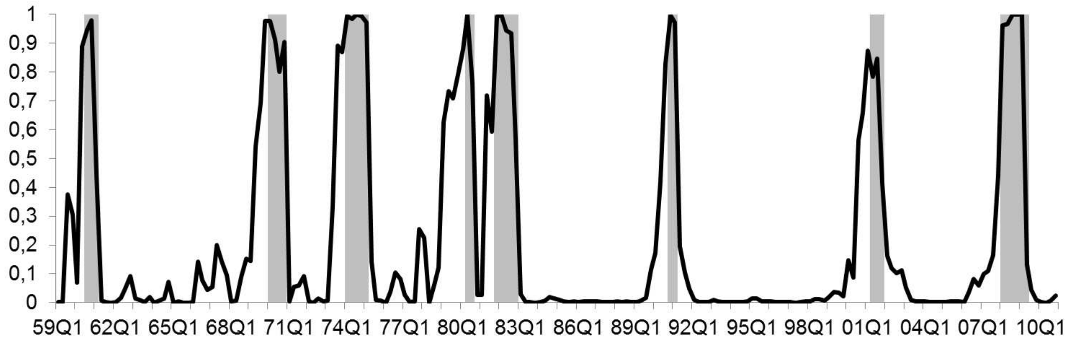
Notes: This table shows the ratios of the RMSFEs relative to the RW model, the QPS and TPI criteria for US GDP growth in the quarters 1990Q1-2010Q4. The TPI criterion is provided for $\lambda = \{0.4, 0.5\}$ and $\tau = 2$. Bold entries outline the best performing model according to each criterion and at each horizon.

Appendix 1: Data sources and descriptions

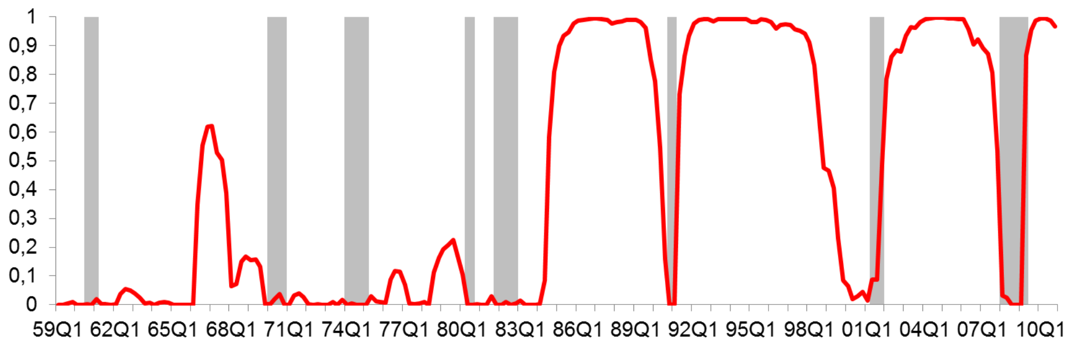
code	variable	transf	delay	datastream code
FM1	M1	6	1	USM1....B
FM2	M2	6	1	USM2....B
FM2DQ	real M2	5	1	USM2....D
FMFBA	monetary base, adjusted for reserve requirement changes	6	0	USM0....B
FMRRA	depository inst reserves:total,adj for reserve req chgs	6	0	USTOTRSAB
FMRNBA	depository inst reserves:nonborrowed, adj for reserve req chgs	6	0	USNBRRSAB
FCLNQ	commercial - industrial loans oustanting in 1996 dollars	6	1	USFCILO.C
FCLBMC	weekly RP LG com'l banks: net change com'l and industrial loans	1	1	USCILNNCB
CCINRV	consumer credit outstanding - nonrevolving	6	1	USCRDCONB
A0M095	Ratio, consumer installment credit to personal income	2	1	USCSCRE%Q
FSPCOM	Standard and Poor's index	5	0	USS PCOM
DJPIN	US Dow Jones Industrials share prince index	5	0	USSHRPRCF
FSDXP	Standard and Poor's: dividend yield	2	0	USSPDIVY
FSPXE	Standard and Poor's: price-earnings ratio	5	0	USSPRPER
FYFF	interest rate: federal funds (effective)	2	0	USFDFUND
CP90	Commercial Paper Rate (AC)	2	0	USI60BC.
FYGM3	interest rate: U.S.treasury bills,SEC MKT,3-MO	2	0	USGBILL3
FYGM6	interest rate: U.S.treasury bills,SEC MKT,6-MO	2	0	USYTB6SM
FYGT1	interest rate: U.S.treasury const maturities,1-YR	2	0	USTRCN1.
FYGT5	interest rate: U.S.treasury const maturities,5-YR	2	0	USTRCN5.
FYGT10	interest rate: U.S.treasury const maturities,10-YR	2	0	USTRCN10
FYAAAC	bond yield: Moody's AAA corporate	2	0	USCRBYLD
FYBAAC	bond yield: Moody's BAA corporate	2	0	USCRBBAA
scp90	cp90-fyff	1	0	(a)
sfygm3	fygm3-fyff	1	0	(a)
sFYGM6	fygm6-fyff	1	0	(a)
sFYGT1	fygt1-fyff	1	0	(a)
sFYGT5	fygt5-fyff	1	0	(a)
sFYGT10	fygt10-fyff	1	0	(a)
sFYAAAC	fyaaac-fyff	1	0	(a)
sFYBAAC	fybaac-fyff	1	0	(a)
EXRUS	US effective exchange rate	5	0	USI..NEUE
EXRSW	exchange rate (Swiss Franc/USD)	5	0	SWXRUSD.
EXRJAN	exchange rate (Yen/USD)	5	0	JPXRUSD.
EXRUK	exchange rate (UK/USD)	5	0	UKXRUSD.
EXRCAN	exchange rate (CAN/USD)	5	0	CNXRUSD.
PWFSA	PPI: finished goods	6	1	USPROPRCE
PWFCSA	PPI: finished consumer goods	6	1	USWPCONFE
PWIMSA	PPI: intermed mat.supplies and components	6	1	USWPINTME
PWCMSA	PPI: crude materials	6	1	USWPCRUDE
PXOIL	average brent oil price	6	1	UKOILBREN
PMCP	NAPM commodity price index	1	1	(b)
PUNEW	CPI: all items	6	1	USCONPRCE
PU83	CPI: apparel and upkeep	6	1	USCPAPPLE
PU84	CPI: transportation	6	1	USCPTRANE
PU85	CPI: medical care	6	1	USCPMEDCE
PUC	CPI-U: commodities	6	1	USCPCOMME
PUCD	CPI-U: durables	6	1	USCPD...E
PUS	CPI-U: services	6	1	USCPSERVE
PUXF	CPI-U: all items less food	6	1	USCPXF..E
PUXHS	CPI-U: all items less shelter	6	1	USCPXHS.E
PUXM	CPI-U: all items less medical care	6	1	USCPXMEDE
GMDC	PCE, implicit price deflator	6	1	USUUBA8BE
GMDCD	PCE, implicit price deflator; durables	6	1	USUND7JGE
GMDCN	PCE, implicit price deflator; non durables	6	1	USUBN11JE
GMDCS	PCE, implicit price deflator; services	6	1	USU4H9R7E

Notes: The column transf gives the transformation of the variables taken in level (code 1), first difference (2), second difference (3), log-level (4), log-first-difference (5) or log-second-difference (6).
(a) Difference between interest rates given above
(b) Source: Institute for Supply Management.

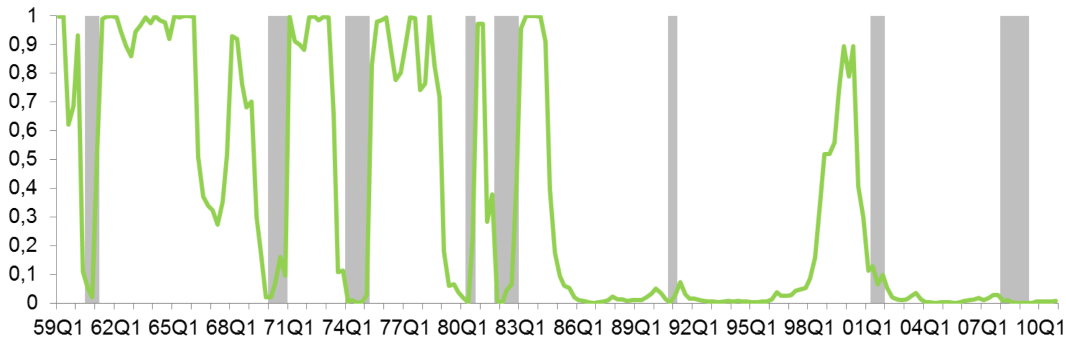
Appendix 2: Smoothed probabilities of the 3-state MS-FaMIDAS model on US GDP



(a) Recession state



(b) Moderate growth (expansion) state



(c) High growth (expansion) state

Notes: This graph plots the smoothed probabilities of the MS-FaMIDAS model (MS3) on US data (1959Q1-2010Q4) and NBER recessions (shaded areas).